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ACHROMATIC COLOURING FOR FOUR COPIES OF BARBELL GRAPH TO FIND ACHROMATIC NUMBER IN BUTTERFLY GRAPH

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ABSTRACT

In this paper, we find the achromatic number for four copies of barbell graph to a butterfly graph. The largest possible number of colours in an achromatic colouring is called the achromatic number and is denoted by $\chi_a(G)$, where G is a finite un directed graph with no loops and multiple edges.

KEYWORDS: *Four Copies of Barbell Graph, Butterfly Graph, Achromatic Number, Vertex Colouring*

INTRODUCTION

Achromatic number is a proper vertex colouring such that each pair of colour classes is adjacent by at least one edge. The largest possible number of colours in an achromatic colouring is called the achromatic number and is denoted by χ_a (G), where G is a finite un directed graph with no loops and multiple edges.

Vertex Colouring

A k-vertex colouring of a graph G, or simply a k- colouring is an assignment of k colours to its vertices. The colouring is proper if no two adjacent vertices are assigned the same colour.

A graph is k- colourable if it has a proper k- colouring.

Edge Colouring

A k-edge colouring of a graph G, or simply a k- colouring is an assignment of k colours to its edges. The colouring is proper if no two adjacent edges are assigned the same colour.

A graph is k-edge colourable if it has a proper k-edge colouring.

Chromatic Number

The chromatic number of a graph G is the least k for which G is k-vertex colourable and it denoted by $\chi(G)$ .A graph G is k-chromatic if $\chi(G)$ = k.

The chromatic number of a graph G is the least k for which G is k-vertex colourable and it denoted by $\chi^{'}(G)$.A graph G is k-chromatic if $\chi^{'}(G)$ = k.

Line Distinguishing Colouring

Let G(V,E) be a graph. A colouring $\phi : V \rightarrow \mathrm{N}$ of the vertices is a line distinguishing colouring iff for every edge (u, v) εE the edge colour (ϕ (u), ϕ(v)) is unique, (i.e). It appears at most once.

Achromatic Colouring and Achromatic Number

The achromatic colouring of a graph is a proper vertex colouring such that each pair of colour classes is adjacent by at least one edge. The largest possible number of colours in an achromatic colouring of a graph G is called the and it is denoted by χ_a(G).

Two Copies of Barbell Graph

Two copies Barbell graph is the simple graph obtained by connecting two copies of a complete graph G_1,$G_2$by a bridge and it is denoted by B(G_1,G_2).

Three Copies of Barbell Graph

Three copies of Barbell graph is the simple graph obtained by connecting three copies of a complete graph G_1, G_2, G_3 by a bridge and it is denoted by B (G_1, G_2, G_3).

Four Copies of Barbell Graph

Four copies of Barbell graph is the simple graph obtained by connecting three copies of a complete graph G_1, G_2, G_3, G_4 by a bridge and it is denoted by B (G_1, G_2, G_3, G_4).

Butterfly Graph

Undirected graphs whose nodes represented as processors and edge represented as inter processor communication links are defined by networks. For example the following figure 1 represents two dimensional butterfly Graph BF (2).

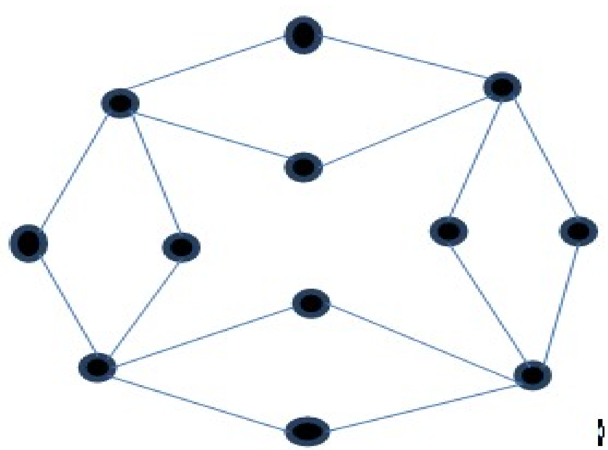

Figure 1: Two Dimensional Butterfly Graph.

Properties of Achromatic Colouring

Table 1: Theorem

Properties	Achromatic Colouring
Lower Bound	In general graphs it is difficult to find lower bound. For particular graphs with large girth (at least 5) admit algorithms with relatively low approximation ratio for the achromatic number. This result gives on the observation that $\chi_a(G) \leq$ m/n for graphs G with n vertices, m edges and girth at least 5.
Upper Bound	In general graphs it is difficult to find upper bound. For a particular case approximating the achromatic number for general or bipartite graphs, the approximation ratio guarantees are just barely sub-linear in the number of vertices.
NP-complete	The problem of achromatic colouring is NP-complete for general graphs.
NP hard	The problem of achromatic colouring is NP-hard for trees, bipartite graphs, interval graphs, bipartite permutation and quasi-threshold graphs

We Find the Following Result for Applying 4 Copies of Barbell Graph to Complete Graph of Butterfly Graph our Definition (8).

Theorem 1

For any complete graph BF(2), Four copies of achromatic colouring is χ_a [B(BF(2),BF(2),BF(2)]=12*4- 4*4=32, n≥ 2

Proof

Let G= B(K_n,K_n,K_n,K_4) be the Barbell graph. By the definition, Four copies of Barbell graph is obtained by connecting three copies of complete graph K_n by a bridge. Let V = {v_1, v_2, v_3......,v_{n-4}} be the vertex set of K^1,W={ w_1, w_2, w_3......,w_{n-4}} be the vertex set of K^2 and X={ x_1, x_2, x_3......,x_{n-4}} be the vertex set of K^3 and Y ={ y_1, y_2,.........y_{n-4}}.

Now the colouring assignments are as follows. Since K^1 contains exactly 'n' vertices (n ≥2) which are mutually adjacent to each other, we should colour the outer vertices using n-4colours and the remaining inner vertices of 'K^1' by n different colours B_1^i,where i=1,2….n. For colouring 'K^2', take from 'K^2' & 'K^3' etc.. We should use outer 'n-4' different colours C_1^i apart from B_1^i and any four colour from B_1^i for inner vertices,1≤ i≤n.

Similarly colour 'K^3', we should use 'n-4' different colours A_1^i apart from C_1^i and any four colour from C_1^i or B_1^i,1≤ i≤n. Thus the number of colours required for =(n-4)+(n-4)+(n-4)=3n-12.

Example: 1

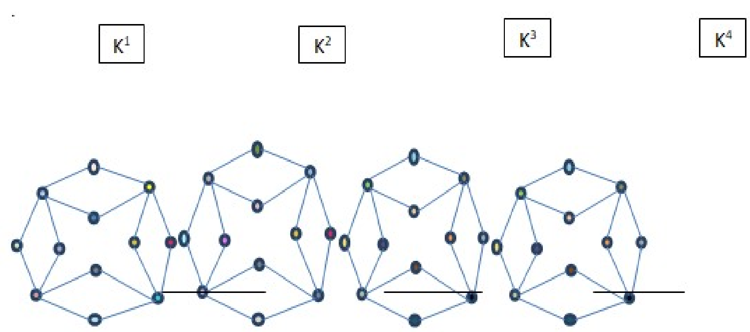

Figure 2: χ_a [B(BF(2),BF(2),BF(2)]=12*4- 4*4=32, n≥ 2.

CONCLUSIONS

We constructed the theorem for four copies of barbell graph and its achromatic number of butterflygraphs. Further research work we were extended to N copies of barbell graph of butterfly graph.

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